Entropy function for non-extremal black holes in string theory

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ABSTRACT: We generalize the entropy function formalism to five-dimensional and four-dimensional non-extremal black holes in string theory. In the near horizon limit, these black holes have BTZ metric as part of the spacetime geometry. It is shown that the entropy function formalism also works very well for these non-extremal black holes and it can reproduce the Bekenstein-Hawking entropy of these black holes in ten dimensions and lower dimensions.

KEYWORDS: Black Holes in String Theory, Black Holes.

Contents

1. Introduction

The black hole attractor mechanism has been an interesting subject over the past years, which states that in a black hole background the moduli fields vary radially and get "attracted" to certain specific values at the horizon which depend only on the quantized charges of the black hole under consideration. As a result, the macroscopic entropy of the black hole is given only in terms of the charges and is independent of the asymptotic values of the moduli. It was first discovered in the context of N=2 extremal black holes [1, 2] and was generalized to theories with higher derivative corrections in [3]. The attractor mechanism for non-supersymmetric black holes was initiated in [4] and discussed more extensively in following papers [5, 6] and [7].

Recently, Sen proposed an efficient way to calculate the entropy of an extremal black hole, which is named as "entropy function" formalism [8]. The main steps can be summarized as follows:

(i) Define a d-dimensional extremal black hole to be such an object that the near horizon geometry is given by $AdS_2 \times S^{d-2}$. Choose a coordinate system in which the AdS_2 part of the metric is proportional to $-r^2 dt^2 + dr^2/r^2$. The black hole background is supported by electric and magnetic fields, as well as various moduli scalar fields.

(ii) Consider a general $AdS_2 \times S^{d-2}$ background characterized by the sizes of AdS_2 and S^{d-2}, the electric and magnetic fields and various scalar fields. Define an entropy function by carrying an integral of the Lagrangian density over S^{d-2} and then taking the Legendre transform of the integral with respect to the parameters e_i denoting the electric fields. The result is a function of the moduli values u_s, the sizes v_1 and v_2 of AdS_2 and S^{d-2}, the electric charges q_i conjugate to e_i, and the magnetic charges p_a.

(iii) For given electric and magnetic charges $\{q_i\}$ and $\{p_a\}$, the values u_s of the scalar fields as well as the sizes v_1 and v_2 are determined by extremizing the entropy function with respect to the variables u_s, v_1 and v_2. Finally the entropy is given by the value of the entropy function at the horizon.

This is a very simple and powerful method to calculate entropy of such kind of black holes. In particular, one can easily obtain the corrections to the entropy due to higher order corrections in the effective Lagrangian. Several related works are given in [9].

As is well known that in string theory, some kinds of black holes can be constructed by putting D-branes together and the Bekenstein-Hawking entropy can be understood by counting the degeneracies of the microstates of such configurations. Such extremal black holes have AdS_3 as part of the near horizon geometry in ten dimensions instead of AdS_2. After dimensional reduction down to lower dimensions, the near horizon geometry turns out to be AdS_2 times a sphere. The entropy function for $D1D5P$ extremal black hole in Type IIB string theory and $D2D6NS5P$ extremal black hole in Type IIA were calculated in [10] and [11], where it was shown that the entropy function formalism could give the correct entropy both in ten dimensions and lower dimensions.

However, for some non-extremal black holes constructed by D-branes, part of the near horizon geometry turns out to be BTZ black hole. Since BTZ black hole is locally equivalent to AdS_3, one expects that the entropy function formalism is also applicable for those black holes. In this paper we show it is indeed the case: after taking the near horizon limit, the entropy function for $5d$ non-extremal black hole can give the Bekenstein-Hawking entropy precisely, while the entropy function for $4d$ non-extremal black hole gives the Bekenstein-Hawking entropy up to a factor, which can be understood via a rescaling transformation relation of entropy function.

The rest of the paper is organized as follows: In section 2 we review the basic properties of the non-extremal black holes. We give a general proof of the entropy function formula for black hole with BTZ as part of its near horizon geometry in section 3. We calculate the entropy function for non-extremal black holes in ten dimensions and lower dimensions in section 4 and section 5 respectively. Finally in the last section 6 we summarize our results and discuss related topics.

2. Non-extremal black holes in string theory

It is known that non-extremal black holes can be constructed by putting D-branes together, so that the entropy can also be obtained by counting the degrees of freedom of the D-brane system after taking near-extremal limit [12]. The microscopic entropy of such kinds of black

holes can also be understood via U-duality [13]. In this section we make a brief review for the salient properties of non-extremal black holes which are necessary in the following calculations. For reviews, see [14].

2.1 The 5d non-extremal black hole

The five dimensional non-extremal black hole can be constructed by using $D1$-branes, $D5$-branes and momentum P, which is a solution of Type IIB supergravity. The effective action is

$$S = \frac{1}{16\pi G_N^{10}} \int d^{10}x \sqrt{-\det g} \left\{ e^{-2\phi}[R + 4(\nabla\phi)^2] - \frac{1}{2}\sum_n \frac{1}{n!}F_n^2 \right\}, \qquad (2.1)$$

where F_n denote the field strengths carried by the D-branes.

The non-extremal black hole metric in $d = 10$ is given as follows in string frame:

$$ds_{10}^2 = f_1(r)^{-\frac{1}{2}}f_5(r)^{-\frac{1}{2}}[-dt^2 + dz^2 + K(r)(\cosh\alpha_m dt - \sinh\alpha_m dz)^2]$$
$$+ f_1(r)^{\frac{1}{2}}f_5(r)^{-\frac{1}{2}}dx_{\parallel}^2 + f_1(r)^{\frac{1}{2}}f_5(r)^{\frac{1}{2}}\left[\frac{dr^2}{1 - K(r)} + r^2 d\Omega_3^2\right],$$
$$e^{-2\phi} = \frac{f_5}{f_1}, \qquad (2.2)$$

where

$$K(r) = \frac{r_H^2}{r^2}, \quad f_1(r) \equiv 1 + \frac{r_1^2}{r^2} = 1 + \frac{r_H^2 \sinh^2\alpha_1}{r^2}, \quad f_5(r) \equiv 1 + \frac{r_5^2}{r^2} = 1 + \frac{r_H^2 \sinh^2\alpha_5}{r^2}, \quad (2.3)$$

and α's are the boost parameters. The $D1$-branes can be viewed as the electric source carrying 3-form electric field strength while the $D5$-branes can be regarded as the magnetic source carrying dual 3-form magnetic field strength. The conserved charges are given by

$$Q_1 = \frac{Vr_H^2}{g_s}\frac{\sinh(2\alpha_1)}{2}, \quad Q_5 = \frac{r_H^2}{g_s}\frac{\sinh(2\alpha_5)}{2}, \quad N = \frac{R_z^2 Vr_H^2}{g_s^2}\frac{\sinh(2\alpha_m)}{2}, \qquad (2.4)$$

where the fundamental string length l_s has been taken to be 1. Here $V = R_5 R_6 R_7 R_8$ where R_i, $i = 5, 6, 7, 8$ denote the radii of the four coordinates in $x_{\parallel}$ and R_z is the radius of the compact dimension z, along which there is a momentum P. The Bekenstein-Hawking entropy is

$$S_{\mathrm{BH}} = \frac{2\pi R_z Vr_H^3}{g_s^2}(\cosh\alpha_1 \cosh\alpha_5 \cosh\alpha_m). \qquad (2.5)$$

To obtain the near horizon geometry, we take the limit

$$r^2 \ll r_{1,5}^2 \equiv r_H^2 \sinh^2\alpha_{1,5}, \qquad (2.6)$$

but we do not demand a similar condition on $r_m \equiv r_H \sinh\alpha_m$. This limit means that α_1, α_5 tend to be very large in the near horizon region, so that $\sinh\alpha_{1,5} \approx \cosh\alpha_{1,5}$. Note that the near horizon limit is just the decoupling limit in the AdS/CFT correspondence. In addition, when $r_H \to 0$ and α_1, $\alpha_5 \to \infty$ while keeping the charges fixed, the non-extremal

black hole turns out to be extremal. After taking such a near horizon limit, the metric becomes

$$ds^2 = -\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\lambda^2\rho^2}dt^2 + \frac{\lambda^2\rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}d\rho^2$$
$$+\rho^2\left(dy - \frac{\rho_+\rho_-}{\lambda\rho^2}dt\right)^2 + \lambda^2 d\Omega_3^2 + \frac{r_1}{r_5}dx_\parallel^2, \tag{2.7}$$

which is of the form $BTZ \times S^3 \times T^4$. Note that here we have made the coordinate transformation

$$\rho^2 \equiv r^2 + \rho_-^2, \quad y \equiv \frac{z}{\lambda} \tag{2.8}$$

and have introduced the parameters

$$\rho_+ \equiv r_H \cosh\alpha_m, \quad \rho_- \equiv r_H \sinh\alpha_m, \quad \lambda^2 \equiv r_1 r_5. \tag{2.9}$$

2.2 The 4d non-extremal black hole

The four dimensional non-extremal black hole can be taken as a non-extremal intersection of $D2$-branes in (z, x_2), $D6$-branes in $(z, x_2, x_3, x_4, x_5, x_6)$, $NS5$-branes in (z, x_3, x_4, x_5, x_6) and momentum P along z, which is a solution of type IIA supergravity with the effective action

$$S = \frac{1}{16\pi G_N^{10}}\int d^{10}x\sqrt{-\det g}\ \left\{e^{-2\phi}\left[R + 4(\nabla\phi)^2 - \frac{1}{3}H^2\right] - G^2 - \frac{1}{12}F'^2\right.$$
$$\left.-\frac{1}{288}\epsilon^{\mu_1\cdots\mu_{10}}F_{\mu_1\mu_2\mu_3\mu_4}F_{\mu_5\mu_6\mu_7\mu_8}B_{\mu_9\mu_{10}}\right\}, \tag{2.10}$$

where the 2-form $G = dA$, 3-form $H = dB$, 4-form $F = dC$ are the field strengths carried by D6-branes, NS5-branes, D2-branes respectively and $F' = F + 2A \wedge H$.

The four dimensional non-extremal black hole metric, written in ten dimensional string frame, is given as follows:

$$ds_{10}^2 = (f_2 f_6)^{-\frac{1}{2}}[-K^{-1}f dt^2 + K(dz + (K'^{-1} - 1)dt)^2]$$
$$+f_5(f_2 f_6)^{-\frac{1}{2}}dx_2^2 + f_2^{\frac{1}{2}}f_6^{-\frac{1}{2}}(dx_3^2 + dx_4^2 + dx_5^2 + dx_6^2)$$
$$+f_5(f_2 f_6)^{\frac{1}{2}}(f^{-1}dr^2 + r^2 d\Omega_2^2),$$
$$e^{-2\phi} = f_5^{-1}f_6^{\frac{3}{2}}f_2^{-\frac{1}{2}}, \tag{2.11}$$

where

$$f_2 \equiv 1 + \frac{r_2}{r} = 1 + \frac{r_H \sinh^2\alpha_2}{r}, \quad f_5 \equiv 1 + \frac{r_5}{r} = 1 + \frac{r_H \sinh^2\alpha_5}{r},$$
$$f_6 \equiv 1 + \frac{r_6}{r} = 1 + \frac{r_H \sinh^2\alpha_6}{r}, \quad K \equiv 1 + \frac{r_K}{r} = 1 + \frac{r_H \sinh^2\alpha_K}{r},$$
$$K'^{-1} = 1 - \frac{q_K}{r}K^{-1} = 1 - \frac{r_H \sinh\alpha_K \cosh\alpha_K}{r}K^{-1}, \quad f = 1 - \frac{r_H}{r}. \tag{2.12}$$

Here the $D2$-branes can be taken as the electric source carrying 4-form electric field strength while the $D6$-branes and the $NS5$-branes can be taken as the magnetic source carrying

dual 2-form and 3-form magnetic field strengths respectively. Then we can obtain the conserved charges

$$Q_2 = \frac{r_H V}{g_s} \sinh 2\alpha_2, \quad Q_5 = r_H R_2 \sinh 2\alpha_5,$$

$$Q_6 = \frac{r_H}{g_s} \sinh 2\alpha_6, \qquad N = \frac{r_H V R_z^2 R_2}{g_s^2} \sinh 2\alpha_K, \qquad (2.13)$$

where the fundamental string length l_s has been set to be 1. Here $V = R_3 R_4 R_5 R_6$ where $R_i, i = 2, 3, 4, 5, 6$ denote the radii of the coordinates x_i and R_z is the radius of the compact dimension z. The Bekenstein-Hawking entropy is

$$S_{\text{BH}} = \frac{8\pi r_H^2 V R_2 R_z}{g_s^2} \cosh \alpha_2 \cosh \alpha_5 \cosh \alpha_6 \cosh \alpha_K. \qquad (2.14)$$

The near horizon geometry can be obtained in a similar way. First we take the near horizon limit, i.e. we require that

$$r \ll r_{2,5,6} \equiv r_H \sinh^2 \alpha_{2,5,6}, \qquad (2.15)$$

but we do not demand a similar condition on $r_K \equiv r_H \sinh^2 \alpha_K$. This limit means that α_2, α_5, α_6 tend to be very large when the near horizon region is approached, so that $\sinh \alpha_{2,5,6} \approx \cosh \alpha_{2,5,6}$. Note that when $r_H \to 0$ and α_2, α_5, $\alpha_6 \to \infty$ while keeping the charges fixed, the non-extremal black hole turns out to be extremal. After taking near horizon limit, the metric becomes

$$ds^2 = -\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\lambda^2 \rho^2} d\tau^2 + \frac{\lambda^2 \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2$$

$$+ \rho^2 \left(dy - \frac{\rho_+ \rho_-}{\lambda \rho^2} d\tau \right)^2 + \frac{\lambda^2}{4} d\Omega_2^2$$

$$+ \frac{r_5}{(r_2 r_6)^{\frac{1}{2}}} dx_2^2 + \left(\frac{r_2}{r_6} \right)^{\frac{1}{2}} \left(dx_3^2 + dx_4^2 + dx_5^2 + dx_6^2 \right), \qquad (2.16)$$

which is of the form $BTZ \times S^2 \times S^1 \times T^4$. Note that here we have made the coordinate transformation

$$\tau = 2\sqrt{r_5} t, \quad y = \frac{z}{(r_2 r_6)^{\frac{1}{4}}}, \quad \rho^2 = r + \rho_-^2 \qquad (2.17)$$

and have introduced the parameters

$$\rho_+^2 \equiv r_H \cosh^2 \alpha_K, \quad \rho_-^2 \equiv r_H \sinh^2 \alpha_K, \quad \lambda^2 \equiv 4 r_5 \sqrt{r_2 r_6}. \qquad (2.18)$$

3. The entropy function formalism: general proof

In this section we will give a detailed derivation of the entropy function for non-extremal black holes with BTZ as part of the near horizon geometry, following [10] and [11]. Note that the entropy function formalism originates from Wald's entropy formula [15], which

requires that the black hole under consideration should have a bifurcate horizon. Then in the entropy function formalism, the entropy of an extremal black hole should be taken as the extremal limit of a non-extremal black hole. So it is natural to expect that the entropy function formalism is also applicable to non-extremal black holes.

A generalized form of Wald formula was proposed in [16], which states that

$$\mathcal{S}_{\mathrm{BH}} = 4\pi \int_H dx_H \sqrt{\det g_H} \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\lambda\rho}} g_{\mu\lambda}^\perp g_{\nu\rho}^\perp, \tag{3.1}$$

where $\mathcal{L}$ is the Lagrangian density, $\det g_H$ is the determinant of the horizon metric and $g_{\mu\nu}^\perp$ denotes the orthogonal metric obtained by projecting onto subspace orthogonal to the horizon. For a metric of the general form

$$ds^2 = g_{tt}dt^2 + g_{yy}dy^2 + 2g_{ty}dtdy + g_{rr}dr^2 + d\vec{x}^2, \tag{3.2}$$

the orthogonal metric is defined as

$$g_{\mu\nu}^\perp = (N_t)_\mu (N_t)_\nu + (N_r)_\mu (N_r)_\nu, \tag{3.3}$$

where N_t and N_r are unit normal vectors to the horizon

$$N_t = \sqrt{\frac{g^{yy}}{g^{tt}g^{yy} - (g^{ty})^2}} \left(1, 0, -\frac{g^{ty}}{g^{yy}}, 0\right), \quad N_r = \left(0, \frac{1}{\sqrt{g^{rr}}}, 0, 0\right). \tag{3.4}$$

Consider the general near horizon metric which has a BTZ part

$$ds^2 = v_1 \Bigg[-\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\lambda^2 \rho^2} d\tau^2 + \frac{\lambda^2 \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2 \tag{3.5}$$
$$+ \rho^2 \left(dy - \frac{\rho_+ \rho_-}{\lambda \rho^2} d\tau\right)^2 \Bigg] + v_2 d\vec{x}^2.$$

The relevant orthogonal metric and Riemann tensor components are given below

$$g_{\tau\tau}^\perp = \frac{(\rho_+^2 + \rho_-^2 - \rho^2)v_1}{\lambda^2}, \quad g_{\tau y}^\perp = -\frac{\rho_+ \rho_- v_1}{\lambda},$$
$$g_{yy}^\perp = \frac{\rho_+^2 \rho_-^2 v_1}{(\rho_+^2 + \rho_-^2 - \rho^2)}, \quad g_{\rho\rho}^\perp = \frac{\lambda^2 \rho^2 v_1}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}, \tag{3.6}$$

$$R_{\tau\rho\tau\rho} = \frac{\rho^2(\rho^2 - \rho_+^2 - \rho_-^2)v_1}{\lambda^2(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)},$$
$$R_{y\rho y\rho} = -\frac{\rho^4 v_1}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)},$$
$$R_{\tau\rho y\rho} = \frac{\rho^2 \rho_+ \rho_- v_1}{\lambda(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}. \tag{3.7}$$

Then the Wald formula (3.1) can be rewritten as

$$\mathcal{S}_{\mathrm{BH}} = \sum_{i=1}^4 \mathcal{S}_i, \tag{3.8}$$

where

$$\begin{aligned}
\mathcal{S}_1 &= 8\pi \int_H dx_H \sqrt{\det g_H} \frac{\partial \mathcal{L}}{\partial R_{\tau\rho\tau\rho}} g_{\tau\tau}^\perp g_{\rho\rho}^\perp \\
&= -8\pi\lambda^2 v_1 \int_H \sqrt{\det g_H} \frac{\partial \mathcal{L}}{\partial R_{\tau\rho\tau\rho}} R_{\tau\rho\tau\rho}, \\
\mathcal{S}_2 &= 8\pi \int_H dx_H \sqrt{\det g_H} \frac{\partial \mathcal{L}}{\partial R_{y\rho y\rho}} g_{yy}^\perp g_{\rho\rho}^\perp \\
&= 8\pi \frac{\lambda^2 \rho_+^2 \rho_-^2 v_1}{\rho^2(\rho^2 - \rho_+^2 - \rho_-^2)} \int_H \sqrt{\det g_H} \frac{\partial \mathcal{L}}{\partial R_{y\rho y\rho}} R_{y\rho y\rho}, \\
\mathcal{S}_3 &= 16\pi \int_H dx_H \sqrt{\det g_H} \frac{\partial \mathcal{L}}{\partial R_{\tau\rho y\rho}} g_{\tau y}^\perp g_{\rho\rho}^\perp \\
&= -16\pi\lambda^2 v_1 \int_H \sqrt{\det g_H} \frac{\partial \mathcal{L}}{\partial R_{\tau\rho y\rho}} R_{\tau\rho y\rho}, \\
\mathcal{S}_4 &= 8\pi \int_H dx_H \sqrt{\det g_H} \frac{\partial \mathcal{L}}{\partial R_{\tau y\tau y}} \left(g_{\tau\tau}^\perp g_{yy}^\perp - (g_{\tau y}^\perp)^2 \right) \\
&= 0.
\end{aligned} \tag{3.9}$$

Next we define a function f as integral of the Lagrangian density over the horizon,

$$f \equiv \int_H dx_H \sqrt{-\det g} \mathcal{L} \tag{3.10}$$

and rescale the Riemann tensor components as proposed in [8],

$$\begin{aligned}
R_{\tau\rho\tau\rho} &\rightarrow \lambda_1 R_{\tau\rho\tau\rho}, \quad R_{\rho y\rho y} \rightarrow \lambda_2 R_{\rho y\rho y}, \\
R_{\tau\rho y\rho} &\rightarrow \lambda_3 R_{\tau\rho y\rho}, \quad R_{\tau y\tau y} \rightarrow \lambda_4 R_{\tau y\tau y}.
\end{aligned} \tag{3.11}$$

It can be seen that the rescaled Lagrangian $\mathcal{L}_\lambda$ behaves as

$$\frac{\partial \mathcal{L}_\lambda}{\partial \lambda_i} = R_{\mu\nu\lambda\rho}^{(i)} \frac{\partial \mathcal{L}_\lambda}{\partial R_{\mu\nu\lambda\rho}^{(i)}}, \quad i = 1, 2, 3, 4 \tag{3.12}$$

Then the rescaled function f_λ satisfies the following relation

$$\left. \frac{\partial f_\lambda}{\partial \lambda_i} \right|_{\lambda_i=1} = v_1 \int_H dx_H \sqrt{\det g_H} R_{\mu\nu\lambda\rho}^{(i)} \frac{\partial \mathcal{L}_\lambda}{\partial R_{\mu\nu\lambda\rho}^{(i)}} \tag{3.13}$$

with no summation on the right hand side for i. Substituting these relations into (3.8) and (3.9), we obtain the following expression for $\mathcal{S}_{\mathrm{BH}}$

$$\mathcal{S}_{\mathrm{BH}} = -2\pi\lambda^2 \left(\frac{\partial f_{\lambda_1}}{\partial \lambda_1} + \frac{\rho_+^2 \rho_-^2}{\rho^2(\rho_+^2 + \rho_-^2 - \rho^2)} \frac{\partial f_{\lambda_2}}{\partial \lambda_2} + \frac{\partial f_{\lambda_3}}{\partial \lambda_3} \right) \bigg|_{\lambda_1=\lambda_2=\lambda_3=1}. \tag{3.14}$$

Furthermore, since the general Lagrangian should be diffeomorphism invariant, the components of the Riemann tensor entering the Lagrangian must be accompanied by the corresponding components of the inverse metric. Thus we have the following relations

$$\begin{aligned}
\lambda_1 R_{\tau\rho\tau\rho} g^{\tau\tau} g^{\rho\rho} &\sim \lambda_1 v_1^{-1}, \quad \lambda_2 R_{\rho y\rho y} g^{yy} g^{\rho\rho} \sim \lambda_2 v_2^{-1}, \\
\lambda_3 R_{\tau\rho y\rho} g^{\tau y} g^{\rho\rho} &\sim \lambda_3 v_3^{-1}, \quad \lambda_4 R_{\tau y\tau y} g^{\tau\tau} g^{yy} \sim \lambda_4 v_4^{-1}.
\end{aligned} \tag{3.15}$$

Assume that the n-form electric field strength $F_{\tau\rho yp\cdots q}$ and m-form magnetic field strength $H^{(m)}$ satisfy $F_{\tau\rho yp\cdots q} = e_1$ and $H^{(m)} = p_a\sqrt{\Omega_m}$ at the horizon, where e_1, p_a are constants to be determined and Ω_m denotes the measure of m-dimensional unit sphere. In the Lagrangian the electric field strength behaves as

$$\sqrt{(g^{\tau\tau}g^{yy} - (g^{\tau y})^2)g^{\rho\rho}g^{pp}\cdots g^{qq}}\, F_{\tau\rho yp\cdots q} \sim e_1 v_1^{-\frac{3}{2}}. \tag{3.16}$$

Note that no other contributions have any dependence on v_1. Then we can rewrite f_λ as a function of scalars, electric and magnetic field strengths

$$f_\lambda(u_s, v_1, v_2, e_1, p_a) = v_1^{\frac{3}{2}} h\Big(u_s, v_2, \lambda_i v_1^{-1}, e_1 v_1^{-\frac{3}{2}}, p_a\Big), \tag{3.17}$$

where h is a general function and the factor $v_1^{\frac{3}{2}}$ comes from $\sqrt{-\det g}$.

Using (3.17), one can easily derive the following equation

$$\sum_{i=1}^{4} \lambda_i \frac{\partial f_{\lambda_i}}{\partial \lambda_i}\Big|_{\lambda_i = 1} = \frac{3}{2}\left(f - e_1 \frac{\partial f}{\partial e_1}\right) - v_1 \frac{\partial f}{\partial v_1}. \tag{3.18}$$

Then the entropy can be reexpressed by substituting (3.18) into (3.14)

$$\mathcal{S}_{\mathrm{BH}} = -2\pi\lambda^2 \left[\frac{3}{2}\left(f - e_1 \frac{\partial f}{\partial e_1}\right) - \frac{\partial f}{\partial \lambda_2} - \frac{\partial f}{\partial \lambda_4} + \frac{\rho_+^2 \rho_-^2}{\rho^2(\rho_+^2 + \rho_-^2 - \rho^2)} \frac{\partial f}{\partial \lambda_2}\right]. \tag{3.19}$$

To simplify the above expression, we have to make use of the following relations, which can be derived from the symmetries of the Lagrangian,

$$\frac{\partial f}{\partial \lambda_1} = \frac{\partial f}{\partial \lambda_2}, \quad \frac{\partial f}{\partial \lambda_3} = -\frac{\rho_+^2 \rho_-^2}{\rho^2(\rho_+^2 + \rho_-^2 - \rho^2)} \frac{\partial f}{\partial \lambda_2}, \quad \frac{\partial f}{\partial \lambda_1} = \frac{\partial f}{\partial \lambda_3} + \frac{\partial f}{\partial \lambda_4}. \tag{3.20}$$

With the help of (3.20), we obtain a simple expression for the entropy

$$\mathcal{S}_{\mathrm{BH}} = \pi\lambda^2\left(e_1 \frac{\partial f}{\partial e_1} - f\right) \equiv \pi\lambda^2 F. \tag{3.21}$$

Thus we have completed the general derivation for the entropy function and the entropy can be obtained by extremizing the entropy function with respect to the moduli

$$\frac{\partial F}{\partial u_s} = 0, \quad \frac{\partial F}{\partial v_i} = 0, \quad i = 1, 2, \tag{3.22}$$

and then substituting the values of the moduli back into F. Note that the relation $q_i = \frac{\partial f}{\partial e_i}$ does not hold any more in the non-extremal case.

Finally, we would like to stress how the entropy function changes if we rescale the coordinates of the background metric. Note that in order to obtain the standard BTZ metric, some of the coordinates have been rescaled in the previous sections, which can be seen from (2.8) and (2.17). Suppose we make the following coordinate rescaling

$$t \to At \tag{3.23}$$

where A is an arbitrary constant. Since the Lagrangian should be diffeomorphism invariant, one can see from the definition of f (3.10) that under the rescaling (3.23), one has

$$f \to Af. \tag{3.24}$$

Thus the entropy function and the entropy behave as

$$F \to AF, \quad \mathcal{S}_{\text{BH}} \to A\mathcal{S}_{\text{BH}}. \tag{3.25}$$

Such transformations will be used in the following sections.

4. Entropy function in ten-dimensional spacetime

We will calculate the entropy function for the two concrete examples presented in section 2 in the present section. We find that for the five-dimensional non-extremal black hole, the result agrees with the Bekenstein-Hawking entropy precisely, while for the four-dimensional non-extremal black hole, the result is different from the Bekenstein-Hawking entropy by a factor, which can be understood according to the arguments given at the end of the last section.

4.1 Case 1: the 5d non-extremal black hole

First we determine the near horizon field configuration in ten dimensional string frame as follows

$$ds^2 = v_1 \Bigg(-\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\lambda^2 \rho^2} dt^2 + \frac{\lambda^2 \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2$$
$$+ \rho^2 \bigg(\frac{1}{\lambda} dz - \frac{\rho_+ \rho_-}{\lambda \rho^2} dt \bigg)^2 \Bigg) + v_2 \bigg(\lambda^2 d\Omega_3^2 + \frac{r_1}{r_5} dx_\parallel^2 \bigg),$$

$$e^{-2\phi} = u_s, \quad F_{t\rho z} = e_1 = \frac{2\rho}{r_1^2} \frac{v_1^{\frac{3}{2}}}{v_2^{\frac{7}{2}}}, \quad G_{\theta\varphi\psi} = 2r_5^2 \sin^2\theta \sin\varphi. \tag{4.1}$$

Using the above field configuration, the effective action (2.1) turns out to be

$$\mathcal{L} = \frac{1}{16\pi G_N^{10}} \Bigg[u_s \frac{6(v_1 - v_2)}{r_1 r_5 v_1 v_2} + \frac{r_1 r_5}{\rho^2 v_1^3} \frac{e_1^2}{2} - \frac{2r_5}{r_1^3 v_2^3} \Bigg] \tag{4.2}$$

and the entropy function becomes

$$F = \frac{V R_z r_1^3 \rho}{2 g_s^2 r_5} \Bigg[u_s \frac{6(v_2 - v_1)}{r_1 r_5 v_1 v_2} + \frac{r_1 r_5}{\rho^2 v_1^3} \frac{e_1^2}{2} + \frac{2r_5}{r_1^3 v_2^3} \Bigg], \tag{4.3}$$

where we have set $l_s = 1$ so that $G_N^{10} = 8\pi^6 g_s^2$. Substituting the value of e_1 and solving the equations

$$\frac{\partial F}{\partial u_s} = 0, \quad \frac{\partial F}{\partial v_i} = 0, \quad i = 1, 2, \tag{4.4}$$

we have

$$u_s = \frac{r_5^2}{r_1^2}, \quad v_1 = 1, \quad v_2 = 1, \tag{4.5}$$

which gives the correct values of the moduli fields. Put the solution (4.5) back into F, we obtain

$$F = \frac{2VR_z}{g_s^2}\rho. \tag{4.6}$$

Furthermore,

$$\begin{aligned}
\mathcal{S}_{\text{BH}} &= \pi\lambda^2 F \mid_{\rho=\rho_+} \\
&= \frac{2\pi VR_z}{g_s^2} r_1 r_5 r_H \cosh\alpha_m \\
&= S_{\text{BH}},
\end{aligned} \tag{4.7}$$

which is just the black hole entropy in the decoupling limit. Note that here we have used the fact that in this limit, $\sinh\alpha_{1,5} \approx \cosh\alpha_{1,5}$.

4.2 Case 2: the 4d non-extremal black hole

The entropy function for four-dimensional black hole can be calculated in a similar way. First we write down the near horizon field configuration in ten dimensional string frame

$$\begin{aligned}
ds^2 = v_1 &\Bigg[-\frac{1}{(r_2 r_6)^{\frac{1}{2}}} \frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\rho^2} dt^2 + \frac{4 r_5 (r_2 r_6)^{\frac{1}{2}} \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2 \\
&\quad + \frac{\rho^2}{(r_2 r_6)^{\frac{1}{2}}} \left(dz - \frac{\rho_+ \rho_-}{\rho^2} dt \right)^2 \Bigg] \\
&+ v_2 \Bigg[r_5 (r_2 r_6)^{\frac{1}{2}} d\Omega_2^2 + \frac{r_5}{(r_2 r_6)^{\frac{1}{2}}} dx_2^2 + \left(\frac{r_2}{r_6} \right)^{\frac{1}{2}} dx_\parallel^2 \Bigg], \\
e^{-2\phi} = u_s, \quad &F_{tz\rho 2} = e_1 = \frac{\rho}{r_2} \frac{v_1^{\frac{3}{2}}}{v_2^{\frac{5}{2}}}, \\
H_{2\theta\varphi} = -\frac{1}{2} r_5 \sin\theta, \quad &G_{\theta\varphi} = -\frac{1}{2} r_6 \sin\theta.
\end{aligned} \tag{4.8}$$

Under the above field configuration, the effective action (2.10) becomes

$$\mathcal{L} = \frac{1}{16\pi G_N^{10}} \Bigg[u_s \left(\frac{4v_1 - 3v_2}{2 r_5 (r_2 r_6)^{\frac{1}{2}} v_1 v_2} - \frac{1}{2 r_5 (r_2 r_6)^{\frac{1}{2}} v_2^3} \right) - \frac{r_6}{2 r_2 r_5^2 v_2^3} + \frac{e_1^2 r_2 r_6}{2 \rho^2 r_5^2 v_1^3 v_2} \Bigg] \tag{4.9}$$

and the entropy function turns out to be

$$F = \frac{4\rho VR_2 R_z}{g_s^2} \frac{r_2 v_2^{\frac{1}{2}}}{2(r_2 r_6)^{\frac{3}{2}} v_1^{\frac{3}{2}}} \Bigg\{ \frac{(r_2 r_6)^{\frac{3}{2}} v_1^3}{r_2 v_2^3} + v_1^2 \Big[r_2^{\frac{1}{2}} r_6^{\frac{3}{2}} v_1 v_2 + r_2 r_5 u_s \Big(v_1 - 4 v_1 v_2^2 + 3 v_2^3 \Big) \Big] \Bigg\}, \tag{4.10}$$

Solving the equations

$$\frac{\partial F}{\partial u_s} = 0, \quad \frac{\partial F}{\partial v_i} = 0, \quad i = 1, 2, \tag{4.11}$$

we arrive at the correct attractor values of the moduli fields

$$u_s = \frac{r_6^{\frac{3}{2}}}{r_2^{\frac{1}{2}} r_5}, \quad v_1 = 1, \quad v_2 = 1. \tag{4.12}$$

Substituting the solutions (4.12) back into F, we obtain

$$F = \frac{4\rho V R_2 R_z}{g_s^2}. \tag{4.13}$$

Finally,

$$\begin{aligned}
\mathcal{S}_{\mathrm{BH}} &= \pi\lambda^2 F \mid_{\rho=\rho_+} \\
&= \frac{16\pi V R_2 R_z}{g_s^2}(r_2 r_6)^{\frac{1}{2}} r_5 \rho_+ \\
&= 2\sqrt{r_5} S_{\mathrm{BH}},
\end{aligned} \tag{4.14}$$

where we have use the fact that $\sinh\alpha_{2,5,6} \approx \cosh\alpha_{2,5,6}$ in the near horizon region and the result does not agree with the Bekenstein-Hawking entropy by a factor $2\sqrt{r_5}$. Note that one has to make a rescaling transformation (2.17) in order to transform the part spanned by coordinates (t,ρ,z) in (4.8) to be a standard BTZ metric. Further note that one has the transformation relation (3.25) due to the rescaling (3.23). Thus the result (4.14) indeed gives us the entropy of 4d black holes in ten dimensional string frame.

5. Entropy function in lower dimensions

It is well known that after dimensional reduction, the extremal black hole in Type II string theory has AdS_2 as part of its near horizon geometry rather than AdS_3. It has already been noticed in [10] and [11] that although the entropy function could give the correct entropy in lower dimensions, not all the moduli fields could take definite values. In this section we first do the dimensional reduction down to six and five dimensions, keeping the BTZ part of the near horizon metric invariant, then we find that the same results for the entropy can be obtained while some of the moduli fields do not take definite values.

5.1 Case 1: the 5d non-extremal black hole

We do the dimensional reduction on $x_\parallel$ and obtain a six-dimensional black string with near horizon geometry $BTZ \times S^3$. The near horizon field configuration

$$\begin{aligned}
ds^2 = v_1 \bigg(&- \frac{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)}{\lambda^2\rho^2}dt^2 + \frac{\lambda^2\rho^2}{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)}d\rho^2 \\
&+ \rho^2\bigg(\frac{1}{\lambda}dz - \frac{\rho_+\rho_-}{\lambda\rho^2}dt\bigg)^2 \bigg) + v_2\lambda^2 d\Omega_3^2,
\end{aligned} \tag{5.1}$$

$$e^{-2\phi} = u_s, \quad e^{2\psi} = u_T$$

$$F_{t\rho z} = e_1 = \frac{2\rho}{u_T r_5^2}\frac{v_1^{\frac{3}{2}}}{v_2^{\frac{3}{2}}}, \quad G_{\theta\varphi\psi} = 2r_5^2\sin^2\theta\sin\varphi, \tag{5.2}$$

where $e^{2\psi}$ stands for the single moduli for T^4.

The six-dimensional effective Lagrangian, which can be obtained by the standard procedure (see e.g. [17]), becomes

$$\mathcal{L} = \frac{1}{16\pi G_N^6}\sqrt{-\det g^{(6)}}e^{2\psi}\bigg\{ e^{-2\phi}\Big[R^{(6)} + 4(\nabla\phi)^2\Big] - \frac{1}{2}\sum_n\frac{1}{n!}F_n^2 \bigg\}, \tag{5.3}$$

where the superscript stands for that the quantities stay in six dimensions. Using the near horizon field configuration and the effective action, the entropy function is expressed as

$$F = \frac{VR_z\rho r_1 r_5}{2g_s^2} v_1^{\frac{3}{2}} v_2^{\frac{3}{2}} u_T \left[u_s \frac{6(v_2 - v_1)}{r_1 r_5 v_1 v_2} + \frac{r_1 r_5}{\rho^2 v_1^3} \frac{e_1^2}{2} + \frac{2r_5}{r_1^3 v_2^3} \right]. \tag{5.4}$$

Solving the equations after substituting the value of e_1 into F,

$$\frac{\partial F}{\partial u_s} = 0, \quad \frac{\partial F}{\partial u_T} = 0, \quad \frac{\partial F}{\partial v_i} = 0, \quad i = 1, 2, \tag{5.5}$$

we obtain

$$v_1 = v_2 = v, \quad u_s = \frac{r_5^2}{r_1^2 v^2}, \quad u_T = \frac{r_1^2}{r_5^2}, \tag{5.6}$$

where v is an arbitrary constant. Finally, after substituting the solutions back into F, we get

$$\begin{aligned} \mathcal{S}_{\mathrm{BH}} &\equiv \pi\lambda^2 F \mid_{\rho = \rho_+} \\ &= \frac{2\pi VR_z r_1 r_5 \rho_+}{g_s} \\ &= S_{\mathrm{BH}}, \end{aligned} \tag{5.7}$$

which is again the entropy of 5d non-extremal black holes.

5.2 Case 2: the 4d non-extremal black hole

Similarly, we do the dimensional reduction on $x^2 \times x_\parallel$ and obtain a five-dimensional black string with near horizon geometry $BTZ \times S^2$. The near horizon field configuration is

$$\begin{aligned} ds^2 = v_1 &\left[-\frac{1}{(r_2 r_6)^{\frac{1}{2}}} \frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\rho^2} dt^2 + \frac{4r_5(r_2 r_6)^{\frac{1}{2}} \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2 \right. \\ &\left. + \frac{\rho^2}{(r_2 r_6)^{\frac{1}{2}}} \left(dz - \frac{\rho_+ \rho_-}{\rho^2} dt \right)^2 \right] + v_2 r_5 (r_2 r_6)^{\frac{1}{2}} d\Omega_2^2, \\ e^{-2\phi} = u_s, &\quad e^{2\psi} = u_T, \quad e^{\frac{\psi_1}{2}} = u_1, \\ F_{tz\rho}^{(5)} = e_1 = \frac{\rho u_1}{u_T} \frac{r_2^{\frac{1}{4}}}{r_5^{\frac{1}{2}} r_6^{\frac{3}{4}}} \frac{v_1^{\frac{3}{2}}}{v_2}, &\quad H_{\theta\varphi}^{(5)} = -\frac{1}{2} r_5 \sin\theta, \quad G_{\theta\varphi} = -\frac{1}{2} r_6 \sin\theta, \end{aligned} \tag{5.8}$$

where $e^{2\psi}$ and $e^{\frac{\psi_1}{2}}$ denote the single moduli for T^4 and S^1 respectively.

The effective Lagrangian in five dimensions can be expressed as

$$\mathcal{L} = \frac{1}{16\pi G_N^5} \sqrt{-\det g^{(5)}} e^{2\psi} e^{\frac{\psi_1}{2}} \left[e^{-2\phi} \left(R^{(5)} - e^{-\psi_1} H^{(5)2} \right) - G^2 - \frac{1}{3} e^{-\psi_1} F^{(5)2} \right], \tag{5.9}$$

where the superscript signifies that the quantities stay in five dimensions and the 2-form magnetic field strength $H^{(5)}$ as well as the 3-form electric field strength $F^{(5)}$ originate from the ten-dimensional field strengths $H_{2\theta\varphi}$ and $F_{tz\rho 2}$. Note that we have omitted the terms

involving the covariant derivatives of the scalar fields because they are set to be constants at the horizon.

We can work out the five-dimensional entropy function by making use of the above effective action and near horizon field configuration

$$F = \frac{4V R_2 R_z \rho r_5^{\frac{3}{2}} (r_2 r_6)^{\frac{1}{4}} v_1^{\frac{3}{2}} v_2 u_T u_1}{g_s^2} \left[u_s \left(\frac{3v_2 - 4v_1}{2r_5 (r_2 r_6)^{\frac{1}{2}} v_1 v_2} + \frac{1}{2u_1^2 r_2 r_6 v_2^2} \right) \right.$$
$$\left. + \frac{r_6}{2r_2 r_5^2 v_2^2} + \frac{e_1^2 (r_2 r_6)^{\frac{1}{2}}}{2u_1^2 \rho^2 r_5 v_1^3} \right]. \quad (5.10)$$

After substituting the value of e_1 we can solve the equations

$$\frac{\partial F}{\partial u_i} = 0, \quad i = s, T, 1, \qquad \frac{\partial F}{\partial v_j} = 0, \quad j = 1, 2, \quad (5.11)$$

and obtain

$$v_1 = v, \qquad v_2 = v,$$
$$u_s = \frac{r_6^{\frac{3}{2}}}{r_5 r_2^{\frac{1}{2}} v}, \quad u_T = \frac{r_2}{r_6}, \quad u_1 = \frac{r_5^{\frac{1}{2}}}{(r_2 r_6)^{\frac{1}{4}} v^{\frac{1}{2}}}, \quad (5.12)$$

where v is an arbitrary constant, once again.

The entropy can be obtained after substituting the solution back into F

$$\mathcal{S}_{\text{BH}} \equiv \pi \lambda^2 F \mid_{\rho = \rho_+}$$
$$= \frac{16\pi V R_2 R_z (r_2 r_6)^{\frac{1}{2}} r_5 \rho_+}{g_s^2}$$
$$= 2\sqrt{r_5} S_{\text{BH}}. \quad (5.13)$$

Here the factor $2\sqrt{r_5}$ appears again. The reason is the same as the one discussed in the previous section.

6. Summary and discussion

The entropy function formalism proposed by Sen is an efficient way to calculate the entropy of a black hole with AdS_2 as part of the spacetime geometry. However, as far as we know, most of the work have been dealing with extremal black holes. In this paper we show that for some non-extremal black holes in string theory with BTZ as part of the near horizon geometry, the entropy function formalism also works very well and can reproduce the Bekenstein-Hawking entropy both in ten dimensions and lower dimensions. Thus our work generalizes the entropy function formalism to certain non-extremal black holes and we expect that it might also work for other non-extremal black objects, such as black p-branes.

We notice that a relevant issue was presented recently in [7], which describes how to apply the entropy function formalism to near-extremal case. They argued that in order to deal with the runaway behavior of the entropy function, one has to introduce a slight

amount of non-extremality on the black hole side. The non-extremality parameter ϵ truncates the infinite throat of AdS_2 in to a finite size, thus the near horizon geometry is no longer $AdS_2 \times S^{d-2}$. But for sufficiently large charges and small ϵ there will be a region in the black hole spacetime where the geometry is approximately $AdS_2 \times S^{d-2}$, and one can use the entropy function formalism to calculate the entropy in this region. However, in our examples the near horizon geometry BTZ do not rely on the near-extremal limit, but the entropy function formalism still works. It would be interesting to study the relations between the two approaches extensively.

Recently, an intuitional explanation of the black hole attractor/non-attractor behavior has been proposed in [6], which states that the attractor/non-attractor behavior is closely related to the near horizon geometry. For extremal black holes with AdS_2 near horizon geometry, the physical distance from a finite radius coordinate r_0 to the horizon turns out to be infinite, while for non-extremal black holes the distance remains finite. It is clear that the infinite physical distance is crucial to allow a scalar field to forget its initial conditions while in non-extremal case the field only has finite "time" until it reaches the horizon. In our examples, the physical distance from a finite radial coordinate ρ_0 to the outer horizon becomes

$$
\begin{aligned}
d &= \int_{\rho_+}^{\rho_0} \sqrt{g_{\rho\rho}} d\rho \\
&= \lambda \log \left(\sqrt{\rho^2 - \rho_+^2} + \sqrt{\rho^2 - \rho_-^2} \right) |_{\rho=\rho_+}^{\rho=\rho_0} \\
&= \lambda \Big[\log \left(\sqrt{\rho_0^2 - \rho_+^2} + \sqrt{\rho_0^2 - \rho_-^2} \right) - \log \left(\sqrt{\rho_+^2 - \rho_-^2} \right) \Big],
\end{aligned}
\tag{6.1}
$$

which turns out to be finite. However, in certain cases, the scalar fields considered here do exhibit some "attractor" behavior, that is, the values at the horizon can be determined by extremizing the entropy function. So it is worth investigating this phenomenon thoroughly.

Acknowledgments

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